Test 2 Mechanics & Relativity

Friday October 6, 2017, 9:00-11:00, Aletta Jacobshal 1

Before you start, read the following:

- There are 4 problems for a total of 40 points.
- *Write your name and student number on each sheet of paper, including the hyperbolic graphs.*
- Make clear arguments and derivations and use correct notation.
- Support your arguments by clear drawings where appropriate.
- Write in a readable manner, illegible handwriting will not be graded.
- *Good luck!*

The Lorentz transformation equations, with $\gamma \equiv 1/\sqrt{1-\beta^2}$:

$$\begin{aligned} t' &= \gamma(t - \beta x) \ , \qquad t = \gamma(t' + \beta x') \ , \\ x' &= \gamma(x - \beta t) \ , \qquad x = \gamma(x' + \beta t') \ , \\ y' &= y \ , \\ z' &= z \ . \end{aligned}$$

The Einstein velocity transformations:

$$\begin{aligned} v'_x &= \frac{v_x - \beta}{1 - \beta v_x} \ , \qquad v'_y = \frac{v_y\sqrt{1-\beta^2}}{1 - \beta v_x} \ , \qquad v'_z = \frac{v_z\sqrt{1-\beta^2}}{1 - \beta v_x} \ , \\ v_x &= \frac{v'_x + \beta}{1 + \beta v'_x} \ , \qquad v_y = \frac{v'_y\sqrt{1-\beta^2}}{1 + \beta v'_x} \ , \qquad v_z = \frac{v'_z\sqrt{1-\beta^2}}{1 + \beta v'_x} \ . \end{aligned}$$

Problem 1 (*20 minutes; 10 points in total*)

Are the following statements true or false?

1 pnt (a) The proper time between two events depends on the worldline between them.

1 pnt (b) $\Delta t \geq \Delta\tau \geq \Delta s$.

1 pnt (c) A moving object's length can be defined as its speed times the time it takes to pass a given point.

1 pnt (d) When in an inertial frame an object's speed approaches the speed of light, its length goes to zero.

1 pnt (e) Fast-than-light influences violate causality in any inertial frame.

1 pnt (f) Because they cannot be causally connected, events separated by a space-like interval do not exist.

1 pnt (g) The "light cone" associated with an event is inertial-frame independent.

1 pnt (h) The relativstic kinetic energy of a particle is always larger than $\frac{1}{2}m|\vec{v}|^2$.

1 pnt (i) The mass of a particle is the same in all inertial frames, even though its speed is not.

1 pnt (j) The energy and the (magnitude of the) momentum of a light quantum ("photon") are always equal.

Problem 2 (*40 minutes; 10 points in total*)

Spaceship Enterprise is floating in space in Federation territory, at rest relative to the border to the Klingon territory, which is 6 (light-)minutes away in the $+x$-direction. Suddenly, a Klingon warship flies past the Enterprise in the direction of the border at a speed of 3/5. Call this event A, and let it define time zero in both the Enterprise and the Klingon reference frames.

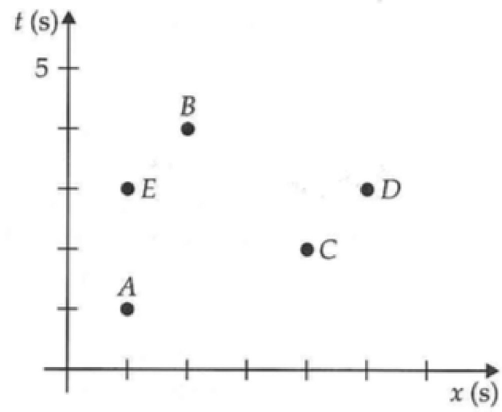
At $t_B = 5$ min according to the Enterprise clocks, the Klingons emit a photon torpedo (event B) that travels at the speed of light back to the Enterprise. It hits the Enterprise and disables it (event C). A bit later, according to the Enterprise radar measurements, the Klingons cross the border into their territory (event D).

4 pnts (a) Draw a two-observer spacetime diagram where the Enterprise and the Klingon warship are the Home and Other Frame, respectively. Draw and label the worldlines (in minutes) of the Enterprise, the Klingon territory border, the Klingon warship, and the photon torpedo. Draw and label events A, B, C, and D as points on the diagram.

3 pnts (b) When does the photon torpedo hit, and when do the Klingons pass into their territory, according to the Enterprise clocks? Answer by reading the times of these events directly from the diagram.

3 pnts (c) According to the Federation-Klingon treaty, it is illegal for a Klingon ship to damage a Federation ship in Federation territory. According to the Klingons, however, the damage to the Enterprise occurred after they had crossed back into Klingon territory. By using your diagram, find out if event C indeed happened after event D in the Klingons' frame. Check your answer with the Lorentz transformation equations.

Problem 3 (*30 minutes; 10 points in total*)



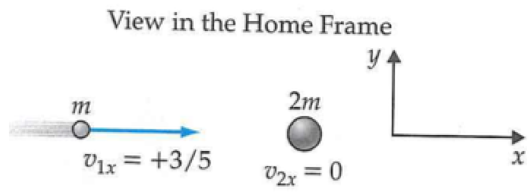
4 pnts (a) For each of the 10 event pairs in the spacetime diagram, classify the spacetime interval between them as timelike, lightlike, or spacelike. Which pairs of events can be causally connected?

3 pnts (b) A flash of laser light is emitted by Earth (event A) and hits a mirror on the Moon (event B). The reflected flash returns to Earth, where it is received (event C). Classify the spacetime interval between A and B, between B and C, and between A and C. Explain your reasoning.

3 pnts (c) A giant gas explosion occurs on the Sun's surface at 12:05 p.m., as measured by an observer in an inertial frame attached to the Sun. At 12:11 p.m., as measured by the same observer, an electricity blackout occurs in Groningen. Can these events be causally connected? Explain.

Problem 4 (*30 minutes; 10 points in total*)

Consider in the Home Frame a one-dimensional elastic collision in which an object of mass m moving at speed $3/5$ in the $+x$-direction hits an object of mass $2m$ at rest.

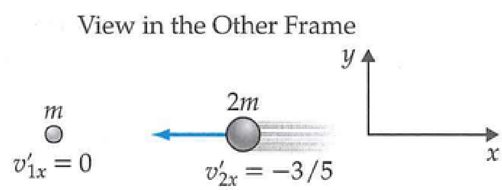


It turns out that conservation of four-momentum implies that the x-velocities of the objects after the collision in the Home Frame are $v_{3x} = -9/41$ for the lighter object and $v_{4x} = 39/89$ for the heavier object.

2 pnts (a) Show that the total x-momentum is indeed conserved in the Home Frame.

2 pnts (b) Show that the total energy is conserved in the Home Frame.

Consider the same collision in the Other Frame, in which the lighter object is initially at rest.



3 pnts (c) Use the Einstein velocity transformation to find the velocities v'_{3x} and v'_{4x} of the objects after the collision.

3 pnts (d) Show that energy and momentum are also conserved in the Other Frame.